Department of Mathematics and Statistics, IISER Kolkata
Mathematics-II
Assignment-3

Instructor: Shirshendu Chowdhury Due Date: 21 February, 2019.

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1. If g is Riemann integrable on $[a, b]$ and $f(x) = g(x)$ except for a point $c \in [a, b]$ Then show that f is Riemann Integrable on $[a, b]$ and $\int_a^b f = . \int_a^b g$.

   Remark: Same result will be true if $f(x) = g(x)$ except for a finite number of points in $[a, b]$ using induction argument.

2. If f is Riemann integrable on $[a, b]$ and P_n is any sequence of tagged partitions of $[a, b]$ such that $||\dot{P}_n|| \longmapsto 0$ as $n \longmapsto \infty$. Then using definition of Riemann Integration (via Riemann sum) show that

$$\int_a^b f = \lim_{n \longmapsto \infty} S(f, \dot{P}_n).$$

3. Let $f : [0, 1] \longmapsto \mathbb{R}$ be defined by
   $f(x) = \frac{1}{2n} \ \ for \ \ x = \frac{1}{n}$ and $f(x) = 0$ elsewhere in $[0, 1]$.
   Show that f is Riemann integrable $[0, 1]$ and $\int_0^1 f = 0$.

4. Evaluate the limits as an integral

$$(i) \lim_{n \longmapsto \infty} [\frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+3n}]$$

$$(ii) \lim_{n \longmapsto \infty} [\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + ... + \sin\frac{n\pi}{n}]$$

$$(iii) \lim_{n \longmapsto \infty} [\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + ... + \frac{n}{n^2+4n^2}]$$

$$(iv) \lim_{n \longmapsto \infty} [(1+\frac{1}{n})(1+\frac{2}{n}).....(1+\frac{n}{n})]^{\frac{1}{n}}$$

$$(v) \lim_{n \longmapsto \infty} [(1+\frac{1}{n^2})(1+\frac{2^2}{n^2})^2.....(1+\frac{n^2}{n^2})^n]^{\frac{1}{n}}$$

5. If f is Riemann Integrable in $[a, b]$. Then show that

   (i) f is bounded on $[a, b]$. i.e there exists a real number $M > 0$ such that

$$|f(x)| \leq M \ \forall x \in [a, b].$$

   (ii) $|\int_a^b f| \leq M(b - a)$ where M is as above.

   Remark : So unbounded function is not Riemann Integrable.

6. Using Fundamental Theorem evaluate

$$(i) \int_{-2}^{2} f \text{ where } f(x) = 3x^2 \cos(\frac{\pi}{x^2}) + 2\pi \sin(\frac{\pi}{x^2}),\ x \neq 0,\ f(0) = 0.$$

$$(ii) \int_{0}^{3} f \text{ where } f(x) = -x, \quad 0 \leq x \leq 1 \quad \text{and} \quad f(x) = x, 1 < x \leq 3$$

$$(iii) \int_{1}^{3} f \text{ where } f(x) = [x], \quad 1 \leq x \leq 3 \ .$$

7. Let $f(x) = x[x],\ x \in [0, 3]$. Using Theorem mentioned in the class show that f is Riemann integrable on $[0, 3]$ and evaluate $\int_0^3 f$.