**Department of Mathematics and Statistics, IISER Kolkata**
**Mathematics-II**
**Assignment-2**

Instructor: Shirshendu Chowdhury  Due Date: 16 February, 2019.

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1. Find the sum (using partial fraction and telescoping sum ) of the following series if they converges

$$(i) \sum_{n=1}^{\infty} \frac{n}{5n+11} \qquad (ii) \sum_{n=0}^{\infty} \frac{3^n+4^n}{5^n}$$

$$(iii) \sum_{n=0}^{\infty} \frac{3^n+5^n}{4^n} \qquad (iv) \sum_{n=1}^{\infty} \sin(\frac{n\pi}{2})$$

$$(v) \sum_{n=1}^{\infty} \frac{1}{n^2+5n+6} \qquad (vi) \sum_{n=1}^{\infty} \frac{1}{n^2+2n}$$

$$(vii) \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} \qquad (viii) \sum_{n=1}^{\infty} \cos n$$

If they diverge justify using Comparison Test (inequality form or Limit form) or $n$-th Term test.

2. Can you give an example of a convergent series $\sum_{n=1}^{\infty} x_n$ and a divergent series $\sum_{n=1}^{\infty} y_n$ such that $\sum_{n=1}^{\infty}(x_n + y_n)$ is convergent? Explain.

3. Use Comparison test (inequality form or Limit form) or $n$- th term test to prove that the following series converge or diverge

$$(i) \sum_{n=1}^{\infty} \frac{n+8}{n^3-5n+7} \qquad (ii) \sum_{n=1}^{\infty} \frac{n+6}{\sqrt{n^3+2}}$$

$$(iii) \sum_{n=1}^{\infty} \frac{\sqrt{5n}-10}{3n+\sqrt{n}} \qquad (iv) \sum_{n=1}^{\infty} \frac{log_e n}{n^2}$$

$$(v) \sum_{n=1}^{\infty}[(n^3+1)^{\frac{1}{3}}-n] \qquad (vi) \frac{1}{1+2}+\frac{1}{1+2^2}+\frac{1}{1+2^3}+...$$

$$(vii) \frac{1}{1+2^{-1}}+\frac{1}{1+2^{-2}}+\frac{1}{1+2^{-3}}+... \qquad (viii) \sin\frac{\pi}{2}+\sin\frac{\pi}{4}+\sin\frac{\pi}{6}+..$$

$$(ix) \frac{1}{1.3}+\frac{2}{3.5}+\frac{3}{5.7}+....... \qquad (x) \frac{2}{1^p}+\frac{3}{2^p}+\frac{4}{3^p}+......$$

4. Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series of positive terms. Prove that convergence of $\sum_{n=1}^{\infty} a_n^2$ and $\sum_{n=1}^{\infty} b_n^2$ implies the convergence of $\sum_{n=1}^{\infty} a_n b_n$.

5. Suppose $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series of positive terms. Let

$$\lim_{n \longmapsto \infty} \frac{a_n}{b_n} = +\infty.$$

If $\sum_{n=1}^{\infty} b_n$ diverges then show that $\sum_{n=1}^{\infty} a_n$ diverges.

6. Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms and

$$b_n = \frac{a_1 + a_2 + ..a_n}{n}.$$

Show that $\sum_{n=1}^{\infty} b_n$ is divergent.