Department of Mathematics and Statistics, IISER Kolkata
Mathematics-II
Assignment-1

Instructor: Shirshendu Chowdhury Due Date: 18 January, 2019.

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1. Using the definition of convergence of sequence (ε and $K(\varepsilon)$) prove that

$$(i) \lim_{n \longmapsto \infty} \frac{n}{n^2+1} = 0 \qquad (ii) \lim_{n \longmapsto \infty} \frac{2n}{n+1} = 2$$
$$(iii) \lim_{n \longmapsto \infty} \frac{3n+1}{2n+5} = \frac{3}{2}. \qquad (iv) \lim_{n \longmapsto \infty} \frac{n^2-1}{2n^2+3} = \frac{1}{2}.$$

2. Let $x_n \geq 0$ for all $n \in \mathbb{N}$ and $\lim_{n \longmapsto \infty} x_n = L$. Then using definition of convergence show that $\lim_{n \longmapsto \infty} \sqrt{x_n} = \sqrt{L}$.

3. Let $\lim_{n \longmapsto \infty} x_n = L$. Then using definition of convergence show that $\lim_{n \longmapsto \infty} |x_n| = |L|$. Is the converse True?

4. Let $\lim_{n \longmapsto \infty} x_n = L$ and $\lim_{n \longmapsto \infty} y_n = L$. Then show that the using definition of convergence that sequence

$$z_n = (z_1, z_2, z_3, z_4.....) = (x_1, y_1, x_2, y_2, ...)$$

converges to L.

5 Using Sandwich Theorem show that

$$(i) \lim_{n \longmapsto \infty} (2^n + 3^n)^{\frac{1}{n}} = 3 \qquad (ii) \lim_{n \longmapsto \infty} \frac{1.3.5....(2n-1)}{2.4.6...2n} = 0.$$

6 Let $\lim_{n \longmapsto \infty} x_n = 0$ and y_n is a bounded sequence. Then show that $\lim_{n \longmapsto \infty} x_n y_n = 0$. Utilise it to prove that

$$\lim_{n \longmapsto \infty} \frac{(-1)^n n}{n^2+1} = 0.$$

7. Find the limits of the following sequences using Sandwich Theorem

$$(i)\ x_n = n^{\frac{1}{n^2}} \qquad (ii)\ y_n = (n!)^{\frac{1}{n^2}}.$$

8. Show that sequence $x_n = \sin(\frac{n\pi}{2})$ is not convergent.

9. Show that

$$(i) \lim_{n \longmapsto \infty} ((2n)^{\frac{1}{n}}) = 1 \qquad (ii) \lim_{n \longmapsto \infty} \frac{n^2}{n!} = 0$$
$$(iii) \lim_{n \longmapsto \infty} \frac{2^n}{n!} = 0.$$