MA 1101 : Mathematics I

Problem 1.

Let $\emptyset \neq D \subseteq \mathbb{R}$, let $c \in D$ and let $f : D \to \mathbb{R}$ be continuous at c with f(c) > 0. Show that, there exists $\delta > 0$ such that

$$f(x) > 0$$
, for all $x \in (c - \delta, c + \delta) \cap D$.

Problem 2.

Let $\emptyset \neq D \subseteq \mathbb{R}$, let $c \in D$ and let $f, g: D \to \mathbb{R}$ be continuous at c. Show that

- (i) f + g is continuous at c.
- (ii) For all $\alpha \in \mathbb{R}$, αf is continuous at c.
- (iii) fg is continuous at c.

(iv) If $g(c) \neq 0$, $\frac{f}{g}$ is continuous at c.

Problem 3.

Let $I \subseteq \mathbb{R}$ be an open interval, let $c \in I$ and let $f, g: I \to \mathbb{R}$ be differentiable at c. Show that

- (i) f + g is differentiable at c and (f + g)'(c) = f'(c) + g'(c).
- (ii) For all $\alpha \in \mathbb{R}$, αf is differentiable at c and $(\alpha f)'(c) = \alpha f'(c)$.
- (iii) fg is differentiable at c and (fg)'(c) = f'(c)g(c) + g'(c)f(c).
- (iv) If $g(c) \neq 0$, $\frac{f}{g}$ is differentiable at c and $\left(\frac{f}{g}\right)'(c) = \frac{f'(c)g(c) f(c)g'(c)}{g(c)^2}$.

Problem 4.

Establish the following inequalities.

- (i) $\frac{x}{1+x} < \ln(1+x) < x$, for all x > 0.
- (ii) $e^x > 1 + x + \frac{x^2}{2}$, for all x > 0.
- (iii) $|\sin x \sin y| \leq |x y|$, for all $x, y \in \mathbb{R}$.