MA 1101 : Mathematics I

Problem 1.
Let $X, Y, Z \neq \emptyset$, let $f : X \to Y$ and let $g : Y \to Z$. Prove that

(i) $g \circ f$ is one-one if f, g are one-one.

(ii) $g \circ f$ is onto if f, g are onto.

(iii) $g \circ f$ is bijective if f, g are bijective.

(iv) g is onto if $g \circ f$ is onto. Is f onto if $g \circ f$ is onto ?

(v) f is one-one if $g \circ f$ is one-one. Is g one-one if $g \circ f$ is one-one ?

(vi) g is one-one if $g \circ f$ is one-one and f is onto.

Problem 2.
Let $W, X, Y, Z \neq \emptyset$ and let $f : W \to X$, $g : X \to Y$, $h : Y \to Z$. Show that

$$(h \circ g) \circ f = h \circ (g \circ f) .$$

Problem 3.
Check whether the following functions are one-one and/or onto.

(i) $f : \mathbb{R} \to \mathbb{R}$, $f(x) := x^2 + x$.

(ii) $f : \mathbb{N} \to \mathbb{N}$, $f(x) := \left[\frac{n+1}{2}\right]$, where $[\cdot]$ denotes the greatest integer function.

(iii) $f : \mathbb{R} \to \mathbb{R}$, $f(x) := x + [x]$.

(iv) $f : \mathbb{R} \to \mathbb{R}$, $f(x) := x - [x]$.

(v) $f : \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f(x) := \frac{x+1}{x-1}$.

(vi) $f : (-1, 1) \to \mathbb{R}$, $f(x) := \frac{x}{1-|x|}$.