MA 1101 : Mathematics I

Problem 1.
Define a relation R on $\mathbb{R} \times \mathbb{R}$ as
$$(x_1, x_2)R(y_1, y_2) \text{ if } x_1 = y_1.$$

 (i) Check that R is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 2.
Define a relation R on $\mathbb{R} \times \mathbb{R}$ as

$$(x_1, x_2)R(y_1, y_2) \text{ if } x_1^2 + x_2^2 = y_1^2 + y_2^2.$$

 (i) Check that R is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 3.
Define a relation R on $\mathbb{N} \times \mathbb{N}$ as
$$(m, n)R(p, q) \text{ if } m + q = n + p.$$

 (i) Check that R is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 4.
Define a relation R on $\mathbb{R} \times \mathbb{R} \setminus \{(0, 0)\}$ as

$$(x_1, x_2)R(y_1, y_2) \text{ if } (y_1, y_2) = \alpha(x_1, x_2), \text{ for some } \alpha \neq 0.$$

 (i) Check that R is an equivalence relation.

 (ii) Identify and draw the equivalence classes.

Problem 5.
Let $n \in \mathbb{N}$ and let X be a set of n elements. Calculate the number of

 (i) relations on X.

 (ii) reflexive relations on X.

 (iii) symmetric relations on X.

 (iv) reflexive and symmetric relations on X.