ES 2201 : Geophysics

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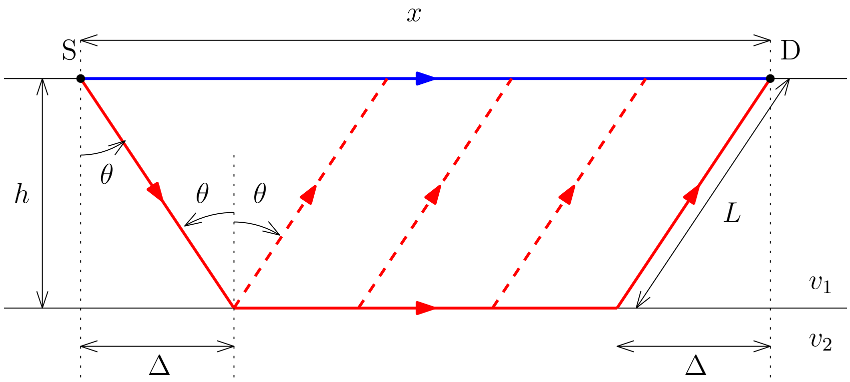
Solution 1.1. A schematic of the path followed by P waves through the earth's interior is presented below.



Note that we have assumed that the waves travel through two layers, both of which are isotropic and homogeneous, with the boundaries perfectly horizontal. The upper layer has a uniform thickness of h. The wave velocities in the upper and lower layers are assumed to be v_1 and v_2 respectively, with $v_2 > v_1$. The blue line denotes the path of the P wave which travels from the source S (which is assumed to be on the surface) directly to the destination D located at a distance x away. The red line shows the path of the wave which strikes the interface of the layers at an angle θ, which turns out to be the critical angle for reflection so that it continues travelling horizontally in the *lower* layer. These wavefronts on the interface emit energy upwards in a fashion symmetrical to that of the incidence, so the wave which reaches the destination D ascends at the same angle θ. The horizontal component of the distance covered during the descent and ascent of the red line is denoted by Δ.

Let the times taken by the blue and red waves to reach D be t_1 and t_2 respectively. Since the blue wave remains in the upper layer throughout, we must have $x = v_1 t_1$, hence

$$\boxed{t_1(x) = \frac{x}{v_1}.}$$

This is the equation of a straight line passing through the origin, with slope $1/v_1$.

For the red wave, we apply Snell's Law to calculate the angle of incidence for which the refracted wave travels horizontally.

$$\frac{\sin\theta}{v_1} = \frac{\sin(\pi/2)}{v_2}, \qquad \sin\theta = \frac{v_1}{v_2}.$$

Now, we use simple trigonometry to obtain the horizontal segments $\Delta = h\tan\theta$ and the lengths of the slanted red segments $L = h/\cos\theta$. The red wave thus travels a distance of $2L$ in the upper layer with speed v_1 and a distance $x - 2\Delta$ in the lower layer with speed v_2, so

$$t_2(x) = \frac{x - 2\Delta}{v_2} + \frac{2L}{v_1} = \frac{x}{v_2} - \left[\frac{2h\tan\theta}{v_2} - \frac{2h}{v_1\cos\theta}\right].$$

This is the equation of a straight line with slope $1/v_2$. Note that Δ and L are fixed independently of x. We also note that the red waves can be observed only when $x \geq 2\Delta$. We can make further progress by noting that if $\sin\theta = v_1/v_2$, then

$$\cos\theta = \frac{\sqrt{v_2^2 - v_1^2}}{v_2}, \qquad \tan\theta = \frac{v_1}{\sqrt{v_2^2 - v_1^2}}.$$

Substituting these expressions, we have

$$t_2(x) = \frac{x}{v_2} - \left[\frac{2hv_1}{v_2\sqrt{v_2^2 - v_1^2}} - \frac{2hv_2}{v_1\sqrt{v_2^2 - v_1^2}}\right] = \frac{x}{v_2} - \frac{2h}{\sqrt{v_2^2 - v_1^2}}\left[\frac{v_1}{v_2} - \frac{v_2}{v_1}\right].$$

Simplifying,

$$\boxed{t_2(x) = \frac{x}{v_2} + \frac{2h}{v_1 v_2}\sqrt{v_2^2 - v_1^2}.}$$

Note that since $v_2 > v_1$, the slope of the $t - x$ curve of the blue line is *more* than that of the red line. On the other hand, the red line has a positive time intercept $t_2(x = 0)$. This means that with increasing x, the blue wave arrives first until a crossover point where the red wave takes over (due to faster travel with v_2). This crossover point x_c can be obtained by setting $t_1 = t_2$, so

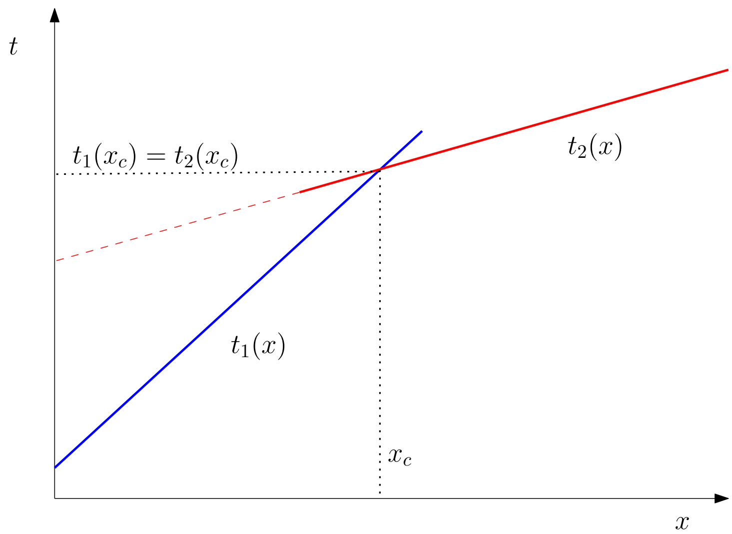
$$\frac{x_c}{v_1} = \frac{x_c}{v_2} + \frac{2h}{v_1 v_2}\sqrt{v_2^2 - v_1^2}, \qquad \frac{v_2 - v_1}{v_1 v_2}x_c = \frac{2h}{v_1 v_2}\sqrt{v_2^2 - v_1^2}.$$

This gives

$$x_c = 2h\frac{\sqrt{v_2^2 - v_1^2}}{v_2 - v_1} = 2h\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}.$$

The time of crossover is

$$t_1(x_c) = t_2(x_c) = \frac{2h}{v_1}\sqrt{\frac{v_2 + v_1}{v_2 - v_1}}.$$



The blue waves are called P_g waves, while the red waves are called P_n waves.

Note: The blue curve observed in reality does not pass through the origin, instead there is a time lag even at $x = 0$, directly above the focus of the earthquake. This is because the focus is typically located at some depth, so the waves take some time to reach the surface. If we had to take this into account in our analysis, the blue curve would not be linear. Supposing the depth of the focus to be f, we see that the distance to the receiving station is $\sqrt{f^2 + x^2}$, so the arrival time is given by

$$t_1'(x) = \frac{\sqrt{f^2 + x^2}}{v_1}.$$

When f/x is small, we can write $\sqrt{1 + f^2/x^2} \approx 1 + f^2/2x^2$, so

$$t_1'(x) \approx \frac{x}{v_1} + \frac{f^2}{2v_1 x}.$$

As x grows large, we get back our simpler expression $t_1'(x) \to x/v_1$.

Solution 1.2

(a) (i) The S wave velocity curve vanishes in the outer core region, and reappears in the inner core.

(ii) The P and S wave velocities seem to increase with depth along with density, except at points of discontinuity where they jump up or down. The major drop in P wave velocity is at the D'' layer as they enter the outer core.

(iii) The density profile has a major jump upwards, also at the D'' layer marking the outer core.

(iv) Both P and S wave velocities rise greatly after exiting the crust, before suffering a dip in the LVZ (Low Velocity Zone). The S wave velocity drop is slightly more pronounced. The density curve shows a similar trend in this region. These changes are comparitively smooth.

(v) The P wave velocities are always higher than S wave velocities in a given zone.

(b) (i) The fact that the S waves vanish in the outer core indicates a liquid outer core. The reappearance of S waves in the inner core suggests that it is solid.

(ii) The smooth changes (increase) in P and S wave velocity indicate that the material in those regions is largely the same. The points of discontinuity mark regions where the material changes abruptly, in terms of physical properties.

(iii) The major jump in density at the outer core indicates a change in composition of the material, and reinforces the idea of a change in state.

(iv) The LVZ is not compositionally too different from the surrounding zones; instead, this zone may have liquid components. It is not completely liquid, but rather is partially melted.

(v) The P wave velocity depends on some additional terms compared to the S waves, which is natural since their physical natures differ (compression waves vs shear waves). The P wave velocity is not merely a scaled up form of the S wave velocity.

(c) (i) S waves are shear waves, which can only properly travel in solids where a restoring force acts on material moved perpendicular to the propagation of the wave. Thus, a liquid outer core would explain why S waves aren't present there.

The presence of S waves in the inner core thus suggests that it is solid. These waves cannot be the same waves generated from the surface, nor from outside the outer core which acts as a barrier. Thus, these must be generated either within the inner core or from the inner-outer core interface, where there may be moving liquid material.

(ii) In a zone where the wave velocities and density vary smoothly (such as in the bulk of the lower mantle or within the outer core), any abrupt changes in material composition would be reflected as an abrupt velocity change. The smooth density increase with depth follows naturally due to the weight of the layers above.

The wave velocities depend on this density too, as

$$v_P = \sqrt{\frac{\kappa + 4\mu/3}{\rho}}, \qquad v_S = \sqrt{\frac{\mu}{\rho}},$$

where κ is the bulk modulus of the material, μ is the shear modulus, and ρ is the density. This would seem to indicate that wave velocity and density ought to have opposing trends – however, the shear modulus μ and bulk modulus κ also increase with depth and have a density dependence, which outweighs the denominator.

The wave velocities can be seen to vary smoothly with physical properties – thus, discontinuities in velocity mark discontinuities in these physical properties.

(iii) Such a large jump at the D'' layer is unlikely to arise purely from a change in composition, where gradual movement of material would perhaps smooth things out. The high density indicates the presence of metal such as iron.

(iv) Again, a smooth velocity and density change indicates a gradual change in the physical properties of the material in that zone. The fact that the S wave velocity drops more than the P wave velocity indicates the presence of some amount of liquid. A complete liquid state would obstruct the passage of S waves in that region. Instead, a partially melted zone would explain the velocity drops.

(v) The fact that the P waves are faster than S waves is evident from their velocity formulae, which we can rearrange as

$$v_P^2 - v_S^2 = \frac{\kappa + \mu/3}{\rho} > 0.$$