
ES1201 : Earth System Processes

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April 24, 2020

Problem 1 List the four layers of Earth's atmosphere. How are they defined? Explain the thermal variation for each layer.

Solution The four layers of the Earth's atmosphere are demarcated by the variation of temperature within them.

- (i) *Troposphere*. This layer extends from the surface to ~ 15 km near the equator and ~ 10 km near the poles. Temperature decreases with increase in altitude.
- (ii) *Stratosphere*. This layer extends from the top of the troposphere to 50 km. Temperature increases with increase in altitude.
- (iii) *Mesosphere*. This layer extends from the top of the stratosphere to 90 km. Temperature decreases with altitude.
- (iv) *Thermosphere*. This layer extends beyond the top of the mesosphere. Temperature increases with increase in altitude.

These different trends in temperature can be explained by the different dominant modes of heating and heat transfer, and different gaseous composition in different layers. We start with the Earth's surface, which is heated by around half of all incoming solar radiation. This energy tries to escape into space as infrared radiation, but cannot penetrate the lower atmosphere easily because of absorption by greenhouse gases and clouds. Thus, the mode of convection dominates heat transfer in the troposphere. Hence, temperature decreases as the distance from the heated Earth surface increases.

Temperatures start increasing again in the stratosphere because of the presence of atmospheric ozone, which peaks at around 30 km. This absorbs ultraviolet radiation, which is available higher up. Maximum heating is thus seen at 50 km. Atmospheric ozone concentration drops off above the stratosphere, so this heating effect decreases, causing a temperature decrease in the mesosphere. In the thermosphere, the absorption of short wave ultraviolet radiation, this time by molecular oxygen, leads to an increase in temperature.

Problem 2 Identify two physical processes by which gases can absorb infrared radiation. Give examples of each process.

Solution Gas molecules can absorb radiation by changing their *rate of rotation* or their *amplitude of vibration*.

A molecule is permitted to have certain discrete rotational frequencies, as governed by quantum mechanics. Each of these is associated with a certain amount of energy. When incident infrared radiation has exactly the right amount of energy, corresponding to the difference between two allowed rotational frequency energies, the molecule absorbs the radiation i.e. the molecule absorbs a single photon of light. Analogously, molecules vibrate in different ways, and have discrete permitted amplitudes of vibration, each associated with an energy level. The molecule can thus 'jump' between energy levels when it absorbs or emits photons with energy exactly equal to the difference between the two energy levels.

For example, water molecules are capable of absorbing radiation with wavelength $12\ \mu\text{m}$ and above, due to their mode of rotation. This effect, when observed in the Earth's atmosphere, is called the H_2O rotation band. Carbon dioxide molecules have three modes of vibration, one of which is a bending mode. This allows the absorption of infrared radiation of wavelength $15\ \mu\text{m}$, which when observed in the Earth's atmosphere is also given its own band.

Problem 3 Explain the physical causes of the greenhouse effect. Why are O_2 and N_2 not greenhouse gases?

Solution The greenhouse effect is caused by the absorption of infrared radiation by certain gases in the atmosphere, called greenhouse gases. These gas molecules absorb IR radiation by processes explained in the preceding answer. Hence, each gas molecule is associated with a certain wavelength ‘window’ where it can efficiently absorb radiation.

Dioxygen and dinitrogen are perfectly symmetric diatomic molecules, and hence show no separation of charge within their molecules. As a result, they do not interact well with electric and magnetic fields. Hence, electromagnetic radiation like infrared radiation pass through unabsorbed, and these molecules do not contribute much to the greenhouse effect.

Note that although carbon dioxide is a symmetric molecule, it is capable of bending while vibrating, which leads to asymmetry.

Problem 4 Given that a 300 K blackbody radiates its peak energy at the wavelength of about 10 mm, at what wavelength would a 900 K blackbody radiate its peak energy?

Solution We use Wien’s Law, which states that the wavelength at which a blackbody emits the maximum amount of radiation flux is inversely proportional to its absolute temperature.

$$\lambda_{max} \propto \frac{1}{T}.$$

Applying this formula directly to the given problem,

$$\lambda_{max,900K} = \frac{T_{300K}}{T_{900K}} \cdot \lambda_{max,300K} = \frac{10}{3} \text{ mm}.$$

Hence, a 900 K blackbody emits its peak energy at a wavelength of approximately 3.3 mm.

Problem 5 Venus and Mars orbit the Sun at average distances of 0.72 AU and 1.52 AU, respectively. What is the solar flux at each planet? Venus has a planetary albedo of 0.8, and Mars has an albedo of 0.22. Determine the effective radiating temperatures of these planets.

Solution We use the inverse square law, which states that the solar flux S received by a planet is inversely proportional to the square of its radial distance r from the Sun.

$$S \propto \frac{1}{r^2}.$$

Using the fact that the solar flux received by the Earth is 1366 W/m^2 , we have

$$\begin{aligned} S_{\text{Venus}} &= \frac{1366}{0.72^2} = 2635 \text{ W/m}^2, \\ S_{\text{Mars}} &= \frac{1366}{1.52^2} = 591 \text{ W/m}^2. \end{aligned}$$

The effective radiating temperature T of a planet can be obtained by assuming them to be blackbodies, then using the Stefan-Boltzmann Law to balance the incoming and outgoing radiation energies. A blackbody with radius R emits radiation at a rate $4\pi R^2 \cdot \sigma T^4$, since it emits radiation proportional to the fourth power of its effective radiating temperature equally in all directions. However, a planet absorbs incoming solar radiation only from one direction, from which it has a projected area of only πR^2 . Hence, the planet absorbs radiation at a rate $\pi R^2 \cdot S$. The albedo A simply means that a portion of this energy, $\pi R^2 \cdot SA$, is reflected back into space. Putting this together,

$$4\pi R^2 \cdot \sigma T^4 = \pi R^2 S - \pi R^2 SA,$$

$$T = \sqrt[4]{\frac{S}{4\sigma}(1 - A)}.$$

Plugging in known values, with the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2/\text{K}^4$,

$$\begin{aligned} T_{\text{Venus}} &= 220 \text{ K}, \\ T_{\text{Mars}} &= 212 \text{ K}. \end{aligned}$$

We note that these are lower than Earth's effective radiating temperature 255 K — Venus has a far higher albedo than Earth, and Mars has a far lower incoming solar flux. Also, these values do not reflect the surface temperatures of these planets. For example, Venus has a significantly higher surface temperature due to the greenhouse gas effect.